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LITERATURE ON INTEGRATION BY NUMERICAL METHODS

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ABSTRACT
The purpose of this paper is to born familiarity with current thinking and research on a integration by numerical methods. Many studies have been conducted over the years to explain about the numerical integration. This paper contains some literature along with some numerical integration methods.

INTRODUCTION
Integrals are important part in mathematical analysis. There are two types of integral - indefinite and definite integral.The first one is called primitive function and it is reverse of derivative. It is also generalization of definite integral. After using the range[a, b]we get definite integral on this range.It can be understood as the area between the function graph and the OX axis, where for the positive values it takes sign plus and for negative minus. Integrals are very useful not only in mathematics analysis, but in the physics calculations too. Numerical analysis is the study of algorithm that use numerical approximation for the problems of the mathematical analysis. Burden L. Richard (2007) finds Numerical analysis applications in all fields of engineering and physical sciences. Aim of the field of the numerical analysis is the analysis of techniques to give approximate but accurate solutions of hard problems . Prasad Devi (2003) reported most numerical analysisists face a problem that can not be solved directly had tried to replace it with a nearby problem that can be solved more easily by the use of interpolation which is very helpful to developing numerical integration and root finding methods. The path to the growth of the integral is a branching one, where comparable discoveries were made concurrently by different people. The history of the method is at present known as integration began with attempts to find the area beneath curves. The fundamentals for the finding of the integral were first laid by Cavalieri, an Italian Mathematician, in around 1635. Cavalier's work centered on the examination that a curve can be measured to be sketched by a moving point and an area to be sketched by a moving line.

HISTORY AND REASONS FOR NUMERICAL INTEGRATION:
The term "numerical integration" first appears in 1915 in the publication A Course in Interpolation and Numeric Integration for the Mathematical Laboratory by David Gibb. Quadrature is a historical mathematical term that means calculating area. Quadrature problems have served as one of the main sources of mathematical analysis. Mathematicians of Ancient Greece, according to the Pythagorean doctrine, understood calculation of area as the process of constructing geometrically a square having the same area .

There are several reasons for carrying out numerical integration.

1. The integrand f(x) may be known only at certain points, such as obtained by sampling. Some embedded systems and other computer applications may need numerical integration for this reason.
2. A formula for the integrand may be known, but it may be difficult or impossible to find an antiderivative that is an elementary function. An example of such an integrand is $f(x) = \exp(-x^2)$, the antiderivative of which (the error function, times a constant) cannot be written in elementary form.
3. It may be possible to find an antivderivative symbolically, but it may be easier to compute a numerical approximation than to compute the anti derivative. That may be the case if the anti derivative is given as an infinite series or product, or if its evaluation requires a special function that is not available.
4. To find the area under an irregular curve we have to integrate the function which represents the curve within the range of the data. Such a function of irregular curve is impossible, but we can measure the value of the function at different points the given range. Using these values we can integrate the unknown function to find the area. This method is widely used in the field of oceanographic, satellite mapping etc.

HISTORY OF TRAPEZOIDAL RULE
A 2016 paper reports that the trapezoid rule was in use in Babylon before 50 BC for integrating the velocity of Jupiter along the ecliptic . The Trapezoid Rule calls for the approximation of area under a curve by fitting trapezoids under the curve and regularly spaced intervals. This method is very common in beginning calculus courses used as a transition into analytical integration. The method uses the outputs of the function as the two

legs of the trapezoid and the specified interval is the height. The area of a trapezoid is one half the heights multiplied by the sum of the two bases.

It is obtained when putting n =1 in general quadrature formulas, the polynomial to be fitted is of degree 1, i.e., a straight line and hence above first order are zero.

HISTORY OF SIMPSON'S RULE

Simpson's Rule, named after Thomas Simpson though also used by Kepler a century before, and was a way to approximate integrals without having to deal with lots of narrow rectangles. .Its strength is that, although rectangles and trapezoids work better for linear functions, Simpson's Rule works quite well on curves. Simpson's Rule is based on the fact that given any three points; you can find the equation of a quadratic through those points. This fact inspired Simpson to approximate integrals using quadratics, as follows.

If you want to integrate f(x) over the interval from a to b,

1. Find f (a), f (b), and f (m) where m is the midpoint of the interval.

2. Find a quadratic P(x) that goes through the same three points. Then, because quadratics is easy to integrate, you could just integrate the quadratic over the interval. It ends up being a very good approximation.

It is obtained when putting n =2 or 3 in general quadrature formulas. It considers second and third order.

History of Weddle's Rule

Thomas Weddle was a mathematician who introduced the weddle surface. He was professor at Royal Military College at Sandhurst. This rule is very helpful to solve multiple integration and gives more accurate solution as compared to any other formulas of quadrature rule especially when you have a working knowledge of programming language like C or MATLAB.

BOOLE'S RULE

$\int_{x_0}^{x_0+nh} f(x)dx = \frac{2h}{45}[7(y_0 + y_n) + 2(y_4 + y_8 + y_{12} + \ldots\ldots + y_{n-4}) + 32(y_1 + y_3 + y_5 + y_7 + \ldots\ldots + y_{n-3} + y_{n-1}) + 12(y_2 + y_6 + y_{10} + \ldots\ldots + y_{n-2})]$

EXTENSION OF QUADRATURE FORMULA FOR n=5.

$\int_{x_0+(n-5)h}^{x_0+nh} f(x)dx = \int_{x_0}^{x_0+nh} f(x)dx = \frac{5h}{288}\{19[(y_0 + y_n) + 2(y_5 + y_{10} + y_{15} + \ldots\ldots + y_{n-5})] + 75(y_1 + y_4 + y_6 + y_9 + y_{11} + y_{14} + \ldots\ldots + y_{n-4} + y_{n-1}) + 50(y_2 + y_3 + y_7 + y_8 + y_{12} + y_{13} + \ldots\ldots + y_{n-3} + y_{n-2})$

EXTENSION OF QUADRATURE FORMULA FOR n=7.

$\int_{x_0}^{x_0+nh} f(x)dx = \frac{7h}{17280}\{ 751[(y_0 + y_n) + 2(y_7 + y_{14} + y_{21} + \ldots\ldots + y_{n-7})] + 3577(y_1 + y_6 + y_8 + y_{13} + y_{15} + y_{20} \ldots\ldots + y_{n-6} + y_{n-1}) + 1323(y_2 + y_5 + y_9 + y_{12} + y_{16} + y_{19} \ldots\ldots + y_{n-5} + y_{n-2}) + 2989 (y_3 + y_4 + y_{10} + y_{11} + y_{17} + y_{18} \ldots\ldots + y_{n-4} + y_{n-3})$.

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